Independent Joint Control

Jee-Hwan Ryu

School of Mechanical Engineering
Korea University of Technology and Education

Introduction to Control

- Control: determining the time history of joint inputs to do a commanded motion
- Control methods are depend on hardware and application
  - Cartesian vs. Elbow
  - Motor with gear reduction vs. High torque motor without gear
  - Continuous path vs. P-to-P
- Control methods have been advanced with the development of complicated hardware
  - The more complicated hardware, the more advanced control methods

Independent Joint Control

- Each Axis -> SISO
- Coupling effect -> disturbance
- Objectives: tracking and disturbance rejection

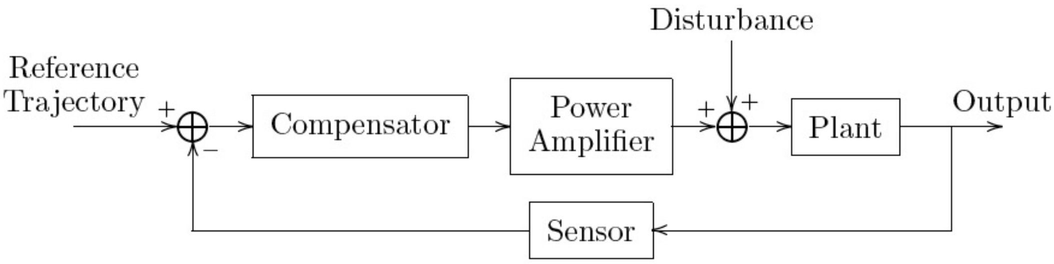
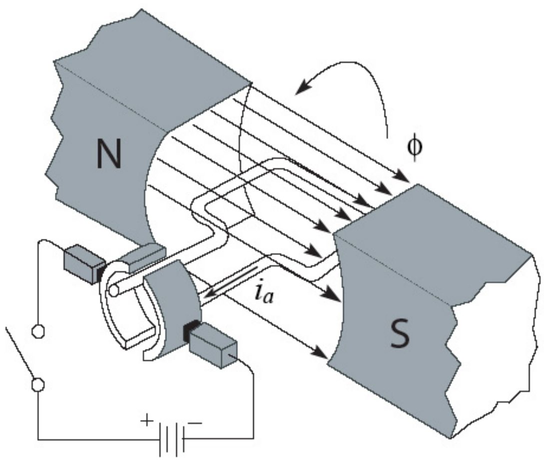


Figure 6.1: Basic structure of a feedback control system. The compensator measures the "error" between a "reference" and a measured "output" and produces signals to the plant that are designed to drive the error to zero despite the presence of disturbances.

Actuator Dynamics

- DC Motor: Simple and easy to use



$$F = i \times \phi$$

$$\tau_m = K_1 \phi i_a$$

$$V_b = K_2 \phi \omega_m$$

Figure 6.2: Principle of operation of a permanent magnet DC motor. The magnitude of the force (or torque) on the armature is proportional to the product of the current and magnetic flux. A **commutator** is required to periodically switch the direction of the current through the armature to keep it rotating in the same direction.

Actuator Dynamics

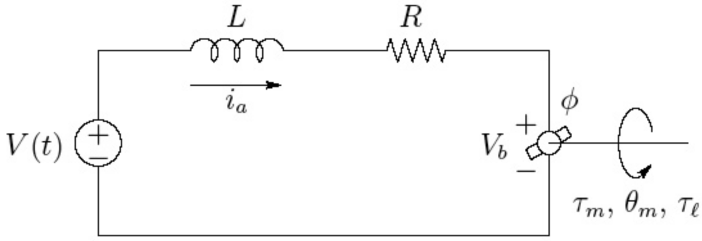


Figure 6.3: Circuit diagram for an armature controlled DC motor. The rotor windings have an effective inductance L and resistance R. The applied voltage V is the control input.

$$L\frac{di_a}{dt} + Ri_a = V - V_b$$

$$\tau_m = K_1\phi i_a = K_m i_a$$

$$V_b = K_2\phi\omega_m = K_b\omega_m = K_b\frac{d\theta_m}{dt}$$

Torque Speed Relation

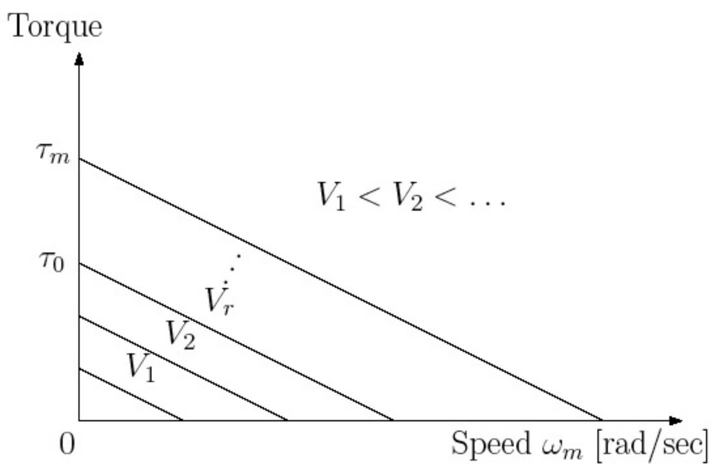


Figure 6.4: Typical torque-speed curves of a DC motor. Each line represents the torque versus speed for a given value of the applied voltage.

Torque Constant

- When motor is stalled

$$V_r = Ri_a = \frac{R\tau_0}{K_m}$$

$$K_m = \frac{R\tau_0}{V_r}$$

Independent Joint Model

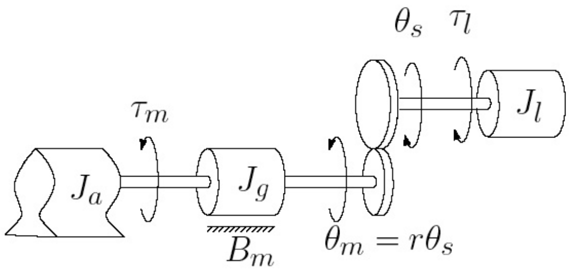


Figure 6.5: Lumped model of a single link with actuator/gear train. J_a, J_g, and J_ℓ are, respectively, the actuator, gear, and load inertias. B_m is the coefficient of motor friction and includes friction in the brushes and gears.

$$J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} = \tau_m - \tau_l / r$$
$$= K_m i_a - \tau_l / r$$

Independent Joint Model

$$(Ls+R)I_a(s)=V(s)-K_b s\Theta_m(s)$$
$$(J_m s^2+B_m s)\Theta_m(s)=K_m I_a(s)-\tau_l(s)/r$$

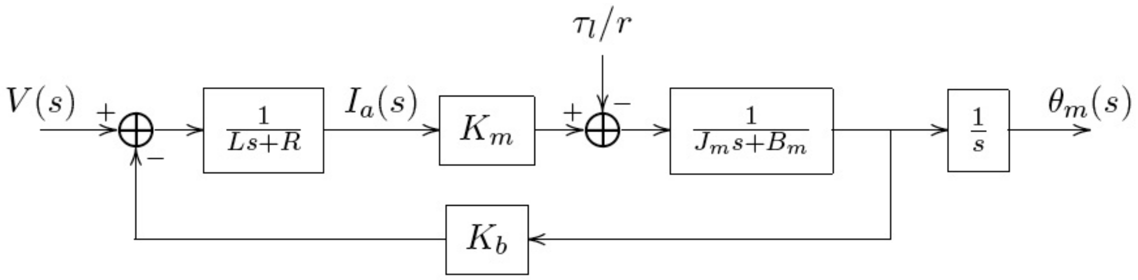


Figure 6.6: Block diagram for a DC motor system. The block diagram represents a third order system from input voltage $V(s)$ to output position $\theta_m(s)$.

Independent Joint Motion

$$\frac{\Theta_m(s)}{V(s)}=\frac{K_m}{s[(Ls+R)(J_m s+B_m)+K_b K_m]},\quad \text{with } \tau_l=0$$
$$\frac{\Theta_m(s)}{\tau_l(s)}=\frac{-(Ls+R)/r}{s[(Ls+R)(J_m s+B_m)+K_b K_m]},\quad \text{with } V=0$$

Effect of the load torque (disturbance) is reduced by the gear reduction

$$\frac{L}{R} << \frac{J_m}{B_m}$$

$$\frac{\Theta_m(s)}{V(s)}=\frac{K_m/R}{s[J_m s+B_m+K_b K_m/R]}$$
$$\frac{\Theta_m(s)}{\tau_l(s)}=\frac{-1/r}{s[J_m s+B_m+K_b K_m/R]}$$

Electrical time constant << Mechanical time constant

Independent Joint Motion

$$J_m\ddot{\theta}_m(t)+(B_m+K_bK_m/R)\dot{\theta}_m(t)=(K_m/R)V(t)-\tau_l(t)/r$$
$$J\ddot{\theta}(t)+B\dot{\theta}(t)=u(t)-d(t)$$

B: effective damping

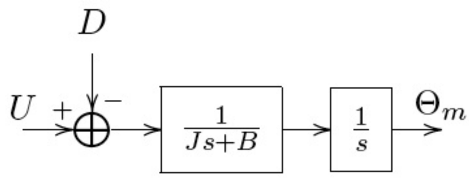


Figure 6.7: Block diagram of the simplified, open-loop system. The disturbance D represents all of the nonlinearities and coupling from the other links.

PD Compensator for Set Point Tracking

- Set point tracking: tracking a constant or step reference

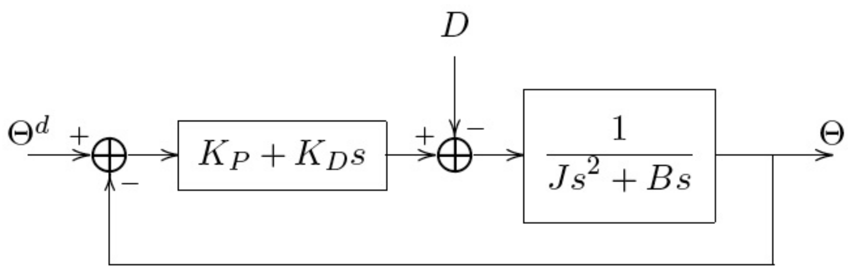


Figure 6.8: The system with PD control. K_P and K_D are the proportional and derivative gains and Θ^d is the desired joint angle to be tracked.

$$\Theta(s)=\frac{K_P+K_Ds}{\Omega(s)}\Theta^d(s)-\frac{1}{\Omega(s)}D(s)$$
$$\Omega(s)=Js^2+(B+K_D)s+K_P$$

PD Compensator

$$E(s) = \Theta^d(s) - \Theta(s)$$
$$= \frac{Js^2 + Bs}{\Omega(s)}\Theta^d(s) + \frac{1}{\Omega(s)}D(s)$$

For a step reference input and a constant disturbance

$$\Theta^d(s) = \frac{\Omega^d}{s}, \quad D(s) = \frac{D}{s}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{D}{K_P}$$

Larger gear reduction and large P-gain can reduce the steady-state error

PD Compensator

- Closed-loop characteristic polynomial

$$s^2 + \frac{(B + K_D)}{J}s + \frac{K_P}{J} = s^2 + 2\zeta\omega s + \omega^2$$
$$K_P = \omega^2 J, \quad K_D = 2\zeta\omega J - B$$

- For robotic applications, critically damped, fastest nonoscillatory response

$$\zeta = 1$$

- ω determined the speed of response

Example 6.1

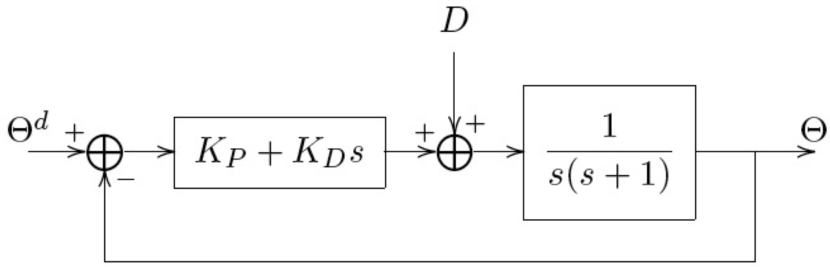


Figure 6.9: Second order system of Example 6.1 with PD Compensator.

Table 6.1: Proportional and derivative gains for the system of Figure 6.9 for various values of natural frequency ω and damping ratio $\zeta = 1$.

Natural Frequency (ω)	Proportional Gain K_P	Derivative Gain K_D
4	16	7
8	64	15
12	144	23

Example 6.1

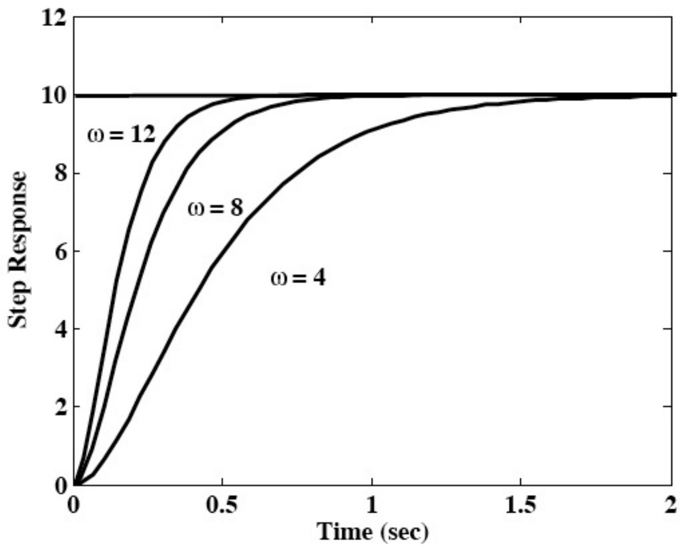


Figure 6.10: Critically damped second order step responses. The rise time decreases for increasing values of ω.

Example 6.2

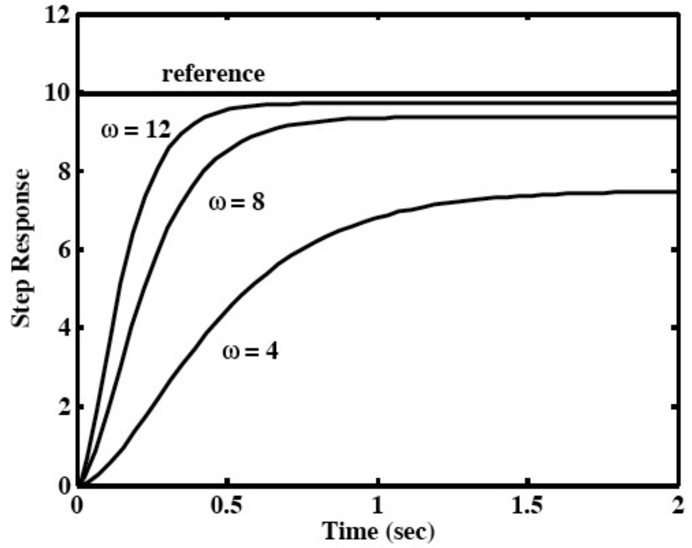


Figure 6.11: Second order system response with PD control and disturbance added. The steady state error decreases for increasing ω.

PID Compensator

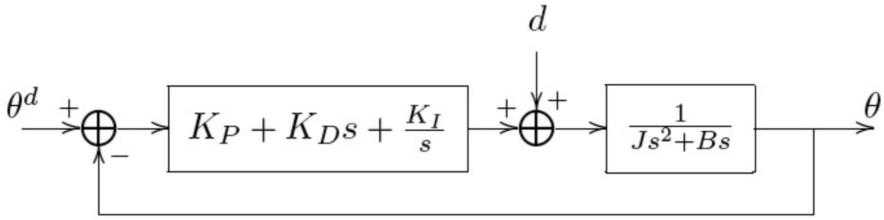


Figure 6.12: Closed-loop system with PID control. The integrator added to the compensator increases the system order from two to three and increases the system type number from 1 to 2.

$$\Theta(s) = \frac{\left(K_D s^2 + K_P s + K_I\right)}{\Omega_2(s)}\Theta^d(s) + \frac{s}{\Omega_2(s)}D(s)$$

$$\Omega_2 = Js^3 + (B + K_D)s^2 + K_P s + K_I$$

Routh's criteria

$$K_I < \frac{(B + K_D)K_P}{J}$$

PID Compensator

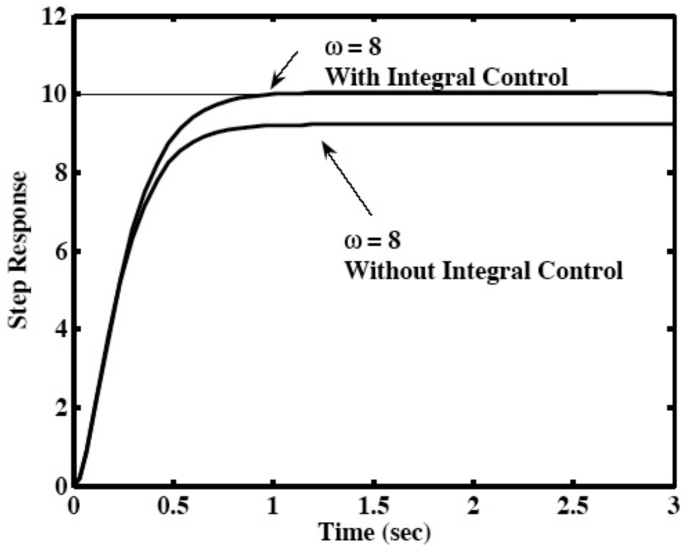


Figure 6.13: Response with integral control action showing that the steady state error to a constant disturbance has been removed.

Design rule-of-thumb for PID Compensator

- First, set K_I =0;
- Desing PD gain to achieve the desired transient behavior
  - Rise time, settling time, etc)
- Design K_I within the limits

The Effect of Saturation and Flexibility

- In theory, arbitrary fast response and arbitrary small steady state error to a constant disturbance can be achieved by simply increasing the gains
- In practice, however, there is a maximum speed of response achievable from the system
- Two major factors
  - Saturation
  - Joint flexibility

Saturation

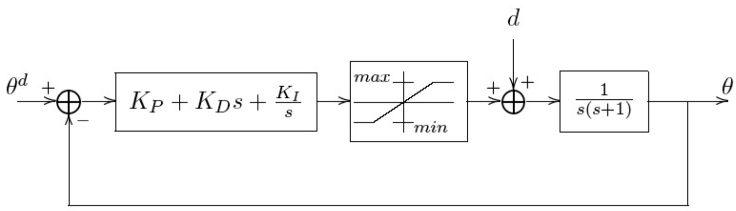


Figure 6.14: Second order system with input saturation limiting the magnitude of the input signal. Increasing the magnitude of the compensator output signal beyond the saturation limit will not increase the input to the plant.

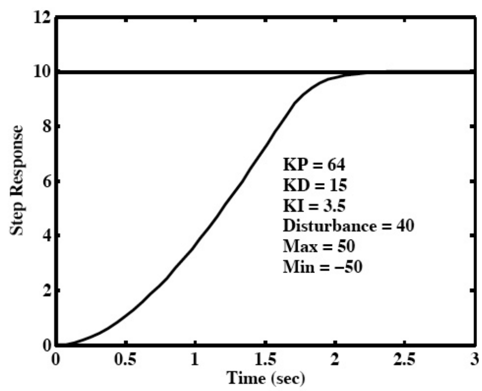


Figure 6.15: Response of the second order system with saturation, disturbance, and PID control. The effect of the saturation is seen in the much slower rise time.

Flexibility

- Should avoid resonant frequency
- Can not increase w arbitrary

Feedforward Control

- To track time-varying trajectories

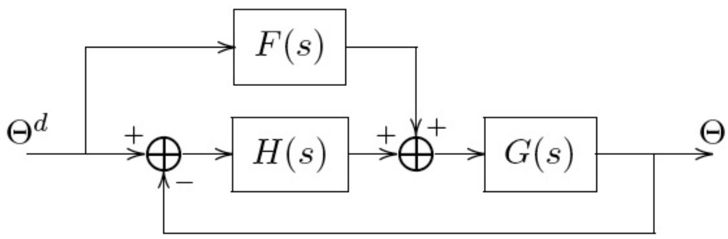


Figure 6.16: Feedforward control scheme. $F(s)$ is the feedforward transfer function which has the reference signal Θ^d as input. The output of the feedforward block is superimposed on the output of the compensator $H(s)$.

$$G(s)=\frac{q(s)}{p(s)},\ H(s)=\frac{c(s)}{d(s)},\ F(s)=\frac{a(s)}{b(s)} \qquad T(s)=\frac{Y(s)}{R(s)}=\frac{q(s)\left(c(s)b(s)+a(s)d(s)\right)}{b(s)\left(p(s)d(s)+q(s)c(s)\right)}$$

Feedforward system and closed-loop system should be stable

Feedforward Control

$$F(s) = 1/G(s)$$

$$R(s) = Y(s)$$

Forward plant is stable -> system is minimum phase

Feedforward Control

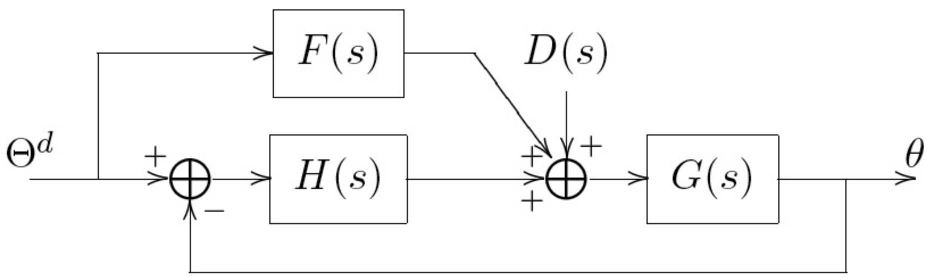


Figure 6.17: Feedforward control with disturbance $D(s)$.

$$E(s) = \frac{q(s)d(s)}{p(s)d(s) + q(s)c(s)} D(s)$$

Feedforward Control

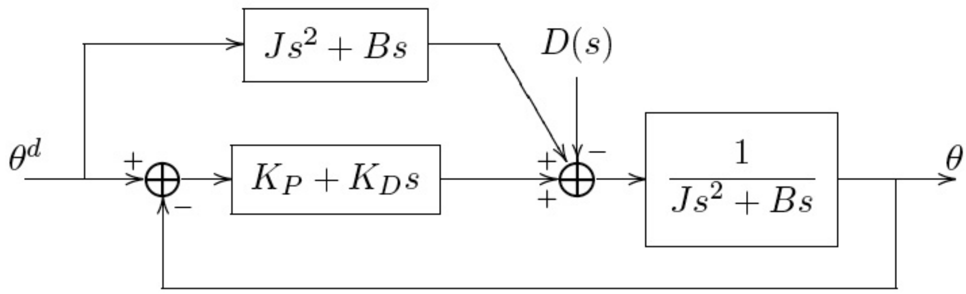


Figure 6.18: Feedforward compensator for the second order system of Section 6.3.

- Differentiation of a actual signal is not required
- Independent of the reference trajectory
- With PID, steady-state error to a step disturbance is zero

$$
\begin{aligned}
V(t) &= J\ddot{\theta}^d + B\dot{\theta}^d + K_D\left(\theta^d - \theta\right) + K_P\left(\theta^d - \theta\right) \\
&= f(t) + K_D\dot{e}(t) + K_P e(t)
\end{aligned}
$$

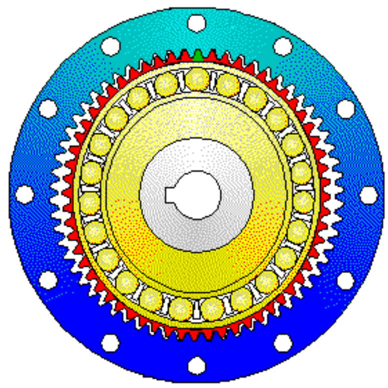
Drive Train Dynamics

- Popular for use in robots due to low backlash, high torque transmission, compact
- Joint flexibility is significant

Figure 6.19: The harmonic drive. The rotation of the elliptical wave generator meshes the teeth of the flexspline and circular spline resulting in low backlash and high torque transmission. (Courtesy of of HD Systems, www.hdsi.net.)

Harmonic Drive

- The Flexspline has two less teeth than the Circular Spline
- The gear ratio is calculated by {#Flexspline Teeth} / {#Flexspline Teeth – #Circular Spline Teeth}.



Drive Train Dynamics

- Flexibility is the limiting factor to the achievable performance in many cases

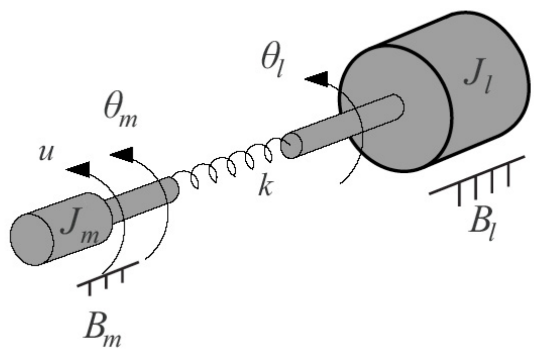


Figure 6.20: Idealized model to represent joint flexibility. The stiffness constant k represents the effective torsional stiffness of the harmonic drive.

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l - \theta_m) = 0$$

$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k(\theta_l - \theta_m) = u$$

Drive Train Dynamics

$$p_l(s)\Theta_l(s) = k\Theta_m(s)$$
$$p_m(s)\Theta_m(s) = k\Theta_l(s) + U(s)$$

$$p_l(s) = J_l s^2 + B_l s + k$$
$$p_m(s) = J_m s^2 + B_m s + k$$

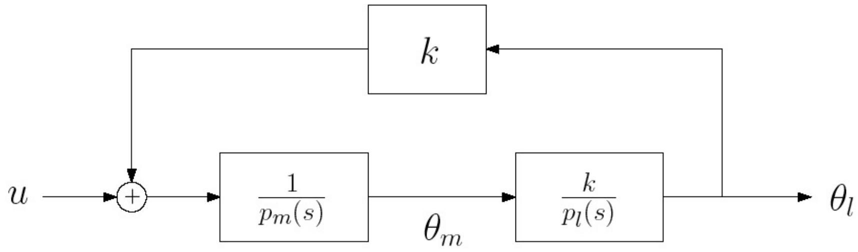


Figure 6.21: Block diagram for the system (6.41) and (6.42).

$$\frac{\Theta_l(s)}{U(s)} = \frac{k}{p_l(s)p_m(s) - k^2}$$

Drive Train Dynamics

- In practice, the stiffness of harmonic drive is large and the damping is small
- Neglect damping

$$J_l J_m s^4 + k(J_l + J_m)s^2$$

- Frequency of imaginary poles increases with increasing joint stiffness
- Difficult to Control

Drive Train Dynamics

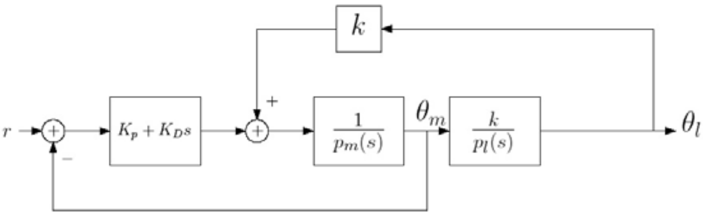


Figure 6.22: PD control with motor angle feedback.

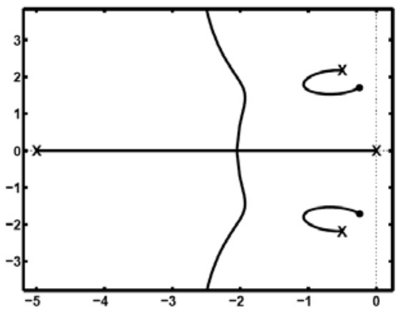


Figure 6.23: Root locus for the system of Figure 6.22.

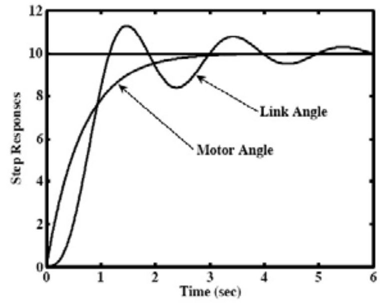


Figure 6.26: Step response — PD control with motor angle feedback. The motor shaft angle, collocated with the motor torque, shows the desired response without overshoot. The motion of the motor shaft excites an oscillation in the load angle, which is effectively outside the feedback loop.

Stable, but undesirable oscillation

Drive Train Dynamics

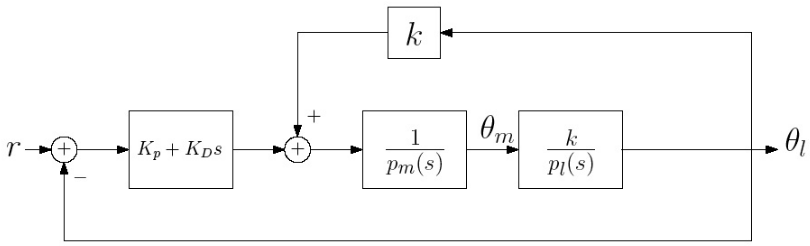


Figure 6.24: PD control with load angle feedback.

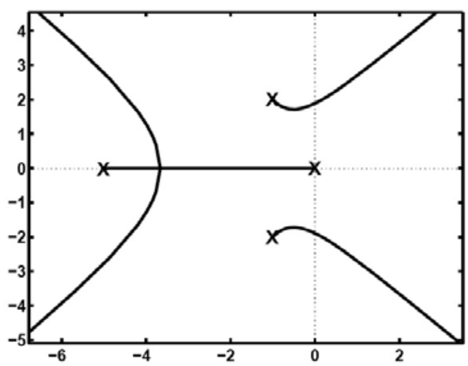


Figure 6.25: Root locus for the system of Figure 6.24.

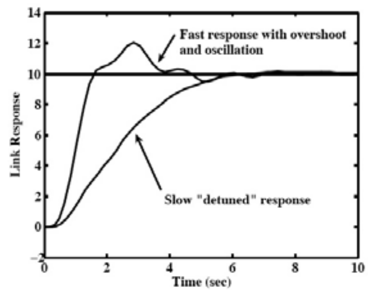


Figure 6.27: Step response — PD control with load angle feedback. The load angle is shown for two different sets of gain parameters. As we know that the system is unstable for large gain, we must effectively "detune" the system for stability, which results in a slower than desired response.

State Space Design

$$x_1 = \theta_l \quad x_2 = \dot{\theta}_l$$
$$x_3 = \theta_m \quad x_4 = \dot{\theta}_m$$

$$\dot{x} = Ax + bu$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_l} & -\frac{B_l}{J_l} & \frac{k}{J_l} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m} & 0 & -\frac{k}{J_m} & \frac{B_m}{J_m} \end{bmatrix}, \quad \mathrm{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix}$$

$$y = c^T x$$
$$c^T = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$G(s) = \frac{\Theta_l(s)}{U(s)} = c^T (sI - A)^{-1} b$$

Poles of the G(s) are eigenvalues of the matrix A

State Feedback Control

$$u(t) = -k^T x + r = \sum_{i=1}^{4} k_i x_i + r$$

Compare with previous PD/PID ?

$$\dot{x} = \left(A - bk^T\right)x + br$$

More free parameters

Controllability

- Definition 6.1: *A linear system is said to be completely controllable, or controllable for short, if for each initial state x(t_0) and each final state x(t_f) there is a control input u(t) that transfer the system from x(t_0) at time t_0 to x(t_f) at time t_f.*
- Lemma 6.1: *A linear system of the form (6.50) is controllable if and only if*

$$\det\begin{bmatrix} b & Ab & A^2b & \cdots & A^{n-1}b \end{bmatrix} \neq 0$$

- Theorem 1: *Let* $\alpha(s) = s^n + \alpha_n s^{n-1} + \cdots + \alpha_2 s + \alpha_1$ *be an arbitrary polynomial of degree n with real coefficients. Then there exists a state feedback control law of the form Eq. (6.55) such that*

$$\det\left(sI - A + bk^T\right) = \alpha(s)$$

if and only if the system (6.50) is controllable.

We may achieve arbitrary closed-loop poles using state feedback

Pole Assignment

- How to choose an appropriate set of closed-loop poles based on the desired performance, the limits on the available torque, etc.
- Optimal Control

$$J = \int_0^\infty \left\{x^T(t)Qx(t) + Ru^2(t)\right\}dt$$

$$u = -k^T x$$

$$k = \frac{1}{R} b^T P$$

$$A^T P + PA - \frac{1}{R} Pb^T bP + Q = 0$$

Observer

- Control law must be a function of all of the states
- Observer: dynamical system (constructed in software), attempts to estimate the full state using the system model and output.

$$\dot{\hat{x}} = A\hat{x} + bu + \ell\left(y - c^T\hat{x}\right)$$

- Assumptions: given the system model, don't know the initial condition

$$e(t) = x - \hat{x}$$
$$\dot{e} = \left(A - \ell c^T\right)e$$

- Observability: the eignevalue of $\left(A - \ell c^T\right)$ can be assigned arbitrary

Observability

- Definition 6.2 *A linear system is completely observable, or observable for short, if every initial state x(t_0) can be exactly determined from measurements of the output y(t) and the input u(t) in a finite time interval.*
- Theorem 2 *the pair (A,c) is observable if and only if*

$$\det\left[c \quad A^T c \quad \cdots \quad {A^T}^{n-1} c\right] \neq 0$$

Seperation Principle

$$\dot{x} = Ax + bu$$
$$u = -k^T \hat{x}$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - bk^T & bk^T \\ 0 & A - \ell c^T \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

- Allows us to separate the design of the state feedback control law from the design of the state estimator
- Place the observer poles to the left of the poles of feedback control law
- Drawbacks
  - Large observer gains can amplify the measurement noise
  - Large gains of state feedback control law can result in saturation of the input
  - Uncertainties in the system parameters
  - nonlinearities