ME1035 ADVANCED ROBOTICS

DYNAMICS

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Introduction

- Relationship between force and motion
- Important in
 - Design of robots
 - Simulation and animation of robot motion
 - Design of control algorithm
- Euler-Lagrange Equation
- Several examples
- Several important properties

The Euler-Lagrange Equations

 A general set of differential equation that describe the time evolution of mechanical systems subject to holonomic constraints

d 0 0

- Two distinct ways of deriving these equations
 - Virtual work
 - Hamilton's principle

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Motivation



Figure 7.1: A particle of mass m constrained to move vertically constitutes a one-degree-of-freedom system. The gravitational force mg acts downward and an external force f acts upward.

$$m\ddot{y} = f - mg$$

$$m\ddot{y} = \frac{d}{dt}(m\dot{y}) = \frac{d}{dt}\frac{\partial}{\partial\dot{y}}\left(\frac{1}{2}m\dot{y}^{2}\right) = \frac{d}{dt}\frac{\partial K}{\partial\dot{y}} \qquad mg = \frac{\partial}{\partial y}(mgy) = \frac{\partial P}{\partial y}$$

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Motivation

$$L = K - P = \frac{1}{2}m\dot{y}^{2} - mgy \qquad Lagrangian$$
$$\frac{\partial L}{\partial \dot{y}} = \frac{\partial K}{\partial \dot{y}} \text{ and } \frac{\partial L}{\partial y} = -\frac{\partial P}{\partial \dot{y}}$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = f \qquad Euler-Lagrange Equation$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = \tau_k; \quad k = 1, \cdots, n \qquad (q_1, \cdots, q_n)$$

Generalized coordinates

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Example 7.1: Single link manipulator



Figure 7.2: Single-link robot. The motor shaft is coupled to the axis of rotation of the link through a gear train which amplifies the motor torque and reduces the motor speed.



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Figure 7.5: A general rigid body has six degrees of freedom. The kinetic energy consists of kinetic energy of rotation and kinetic energy of translation.

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Figure 7.6: A rectangular solid with uniform mass density and coordinate frame attached at the geometric center of the solid.

Two-links Cartesian Manipulators



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Figure 7.7: Two-link planar Cartesian robot. The orthogonal joint axes and linear joint motion of the Cartesian robot result in simple kinematics and dynamics.

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Figure 7.8: Two-link revolute joint arm. The rotational joint motion introduces dynamic coupling between the joints. Planar Elbow Manipulator with Remotely Driven Link

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Figure 7.9: Two-link revolute joint arm with remotely driven link. Because of the remote drive the motor shaft angles are not proportional to the joint angles.

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Planar Elbow Manipulator with Remotely Driven Link



Figure 7.10: Generalized coordinates for the robot of Figure 6.4.

Five-Bar Linkage



Figure 7.11: Five-bar linkage.

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Properties of Robot Dynamic Equations

- Fortunately, robot dynamic equations contain some important structural properties
 - Skew symmetry property
 - Passivity property
 - Linearity-in-the-parameters property
 - Global bounds of inertia matrix for revolute joint robots
- Good advantage for developing control algorithms