Introduction

- Relationship between force and motion
- Important in
  - Design of robots
  - Simulation and animation of robot motion
  - Design of control algorithm
- Euler–Lagrange Equation
- Several examples
- Several important properties
The Euler-Lagrange Equations

- A general set of differential equations that describe the time evolution of mechanical systems subject to holonomic constraints
- Two distinct ways of deriving these equations
  - Virtual work
  - Hamilton's principle

Motivation

Figure 7.1: A particle of mass $m$ constrained to move vertically constitutes a one-degree-of-freedom system. The gravitational force $mg$ acts downward and an external force $f$ acts upward.

\[
m\ddot{y} = f - mg
\]

\[
m\ddot{y} = \frac{d}{dt} (m\dot{y}) = \frac{d}{dt} \left( \frac{d}{\hat{y}} \left( \frac{1}{2} m\dot{y}^2 \right) \right) = \frac{d}{dt} \frac{\partial K}{\hat{y}}
\]

\[
mg = \frac{\partial}{\partial y} (mg\dot{y}) = \frac{\partial P}{\partial y}
\]
Motivation

\[ L = K - P = \frac{1}{2} my^2 - mgy \quad \text{Lagrangian} \]

\[ \frac{\partial L}{\partial y} = \frac{\partial K}{\partial y} \quad \text{and} \quad \frac{\partial L}{\partial \dot{y}} = -\frac{\partial P}{\partial \dot{y}} \]

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = f \quad \text{Euler-Lagrange Equation} \]

\[ \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_k} - \frac{\partial L}{\partial q_k} = \tau_k; \quad k = 1, \ldots, n \quad (q_1, \ldots, q_n) \quad \text{Generalized coordinates} \]

Example 7.1: Single link manipulator

Figure 7.2: Single-link robot. The motor shaft is coupled to the axis of rotation of the link through a gear train which amplifies the motor torque and reduces the motor speed.
Kinetic Energy of a Rigid Object

Figure 7.5: A general rigid body has six degrees of freedom. The kinetic energy consists of kinetic energy of rotation and kinetic energy of translation.

Example 7.2: Uniform Rectangular Solid

Figure 7.6: A rectangular solid with uniform mass density and coordinate frame attached at the geometric center of the solid.
Two-links Cartesian Manipulators

Figure 7.7: Two-link planar Cartesian robot. The orthogonal joint axes and linear joint motion of the Cartesian robot result in simple kinematics and dynamics.

Planar Elbow Manipulator

Figure 7.8: Two-link revolute joint arm. The rotational joint motion introduces dynamic coupling between the joints.
Planar Elbow Manipulator with Remotely Driven Link

Figure 7.9: Two-link revolute joint arm with remotely driven link. Because of the remote drive the motor shaft angles are not proportional to the joint angles.

Planar Elbow Manipulator with Remotely Driven Link

Figure 7.10: Generalized coordinates for the robot of Figure 6.4.
Properties of Robot Dynamic Equations

- Fortunately, robot dynamic equations contain some important structural properties
  - Skew symmetry property
  - Passivity property
  - Linearity-in-the-parameters property
  - Global bounds of inertia matrix for revolute joint robots

- Good advantage for developing control algorithms