Dynamics of Haptic and Teleoperation Systems

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Teleoperation System Overview
Dynamic Model of Teleoperation Systems

1. Mechanical Model
2. Mechanical and Electrical Analogy
3. Electrical Model
4. Understanding of the behavior of Teleoperation Systems

One-DOF Schematic Diagram

The dynamics of the master arm and slave arm is given by the following equations

Master: \( m_m \ddot{x}_m + b_m \dot{x}_m = \tau_m + f_m \)

Slave: \( m_s \ddot{x}_s + b_s \dot{x}_s = \tau_s - f_s \)
### Definition of Parameters

Master:  \[ m_m \ddot{x}_m + b_m \dot{x}_m = \tau_m + f_m \]

Slave:  \[ m_s \ddot{x}_s + b_s \dot{x}_s = \tau_s - f_s \]

- \( x_s \): displacement of master arm
- \( x_s \): displacement of slave arm
- \( m_m \): mass coefficient of the master arm
- \( b_m \): viscous coefficient of the master arm
- \( m_s \): mass coefficient of the slave arm
- \( b_s \): viscous coefficient of the slave arm
- \( f_m \): force that the operator applies to the master arm
- \( f_s \): force that the slave arm applies to the environment
- \( \tau_m \): actuator driving forces of master
- \( \tau_s \): actuator driving forces of slave

### Dynamics of the Environment

The dynamics of the environment interacting with the slave arm is modeled by the following linear system:

\[ f_s = m_e \ddot{x}_s + b_e \dot{x}_s + k_e x_s \]

where
- \( m_e \): mass coefficient of the environment
- \( b_e \): damping coefficient of the environment
- \( k_e \): stiffness coefficient of the environment

The displacement of the object is represented by \( x_s \) because the slave arm is assumed to be rigidly attached with the environments or slave arm firmly grasping the environments, in such a way that it may not depart from the object, once the slave arm contact the environments.
Dynamics of the Operator

It is also assumed that the dynamics of the operator can be approximately represented as a simple spring-damper-mass system

\[ f_{op} - f_m = m_{op} \ddot{x}_m + b_{op} \dot{x}_m + k_{op} x_m \]

where
- \( m_{op} \): mass coefficient of the operator
- \( b_{op} \): damping coefficient of the operator
- \( k_{op} \): stiffness coefficient of the operator
- \( f_{op} \): force generated by the operator’s muscles

The displacement of the operator is represented by \( x_m \) because it is assumed that the operator is firmly grasping the master arm and operator never releases the master arm during the operation.

Master/Operator Cooperative System
Slave/Environment Cooperative System

\[
x_s \rightarrow m_e s^2 + b_e s + k_e
\]

\[
\frac{1}{m_s s^2 + b_s s} \rightarrow f_s \leftarrow \tau_s
\]

Total System

\[
x_m \rightarrow m_{op} s^2 + b_{op} s + k_{op}
\]

\[
\frac{1}{m_m s^2 + b_m s} \rightarrow f_{op} \leftarrow f_m \rightarrow \tau_m
\]

\[
x_s \rightarrow m_e s^2 + b_e s + k_e
\]

\[
\frac{1}{m_s s^2 + b_s s} \rightarrow f_s \leftarrow \tau_s
\]
Let’s Do This

- Simulate dynamics behavior
- How to make stable simulation?


Constitutive Relation

The environment defines a “constitutive relation,” a relation between force and position or one of its derivatives.

Examples:

- Spring $F = K \cdot x$
- Damper $F = B \cdot \dot{x}$
- Inertia $F = M \cdot \ddot{x}$
Electrical Analogy

These relations for mechanical systems are directly analogous to similar relations for electrical systems.

<table>
<thead>
<tr>
<th>Electrical Component</th>
<th>Mechanical Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitor $V$</td>
<td>Force $F$</td>
</tr>
<tr>
<td>$\int idt$</td>
<td>$m\ddot{x} + b\dot{x} + kx$</td>
</tr>
<tr>
<td>$1/C$</td>
<td>$i$</td>
</tr>
</tbody>
</table>

Resistor $V$ to $R$ and Inductor $V$ to $L$.

Force $\leftrightarrow$ Voltage (effort)
Velocity $\leftrightarrow$ Current (flow)

Example 1

Convert an environment defined by the mechanical system

$$F = m\ddot{x} + b\dot{x} + kx$$

to the equivalent electrical circuit. We can make the substitutions

$$V \leftrightarrow F, \ i \leftrightarrow \dot{x}$$

giving

$$V = m\frac{di}{dt} + bi + k\int_0^t idt$$

The parameters $m$, $b$, $k$ correspond to the electrical parameters

$$m \leftrightarrow L, \ b \leftrightarrow R, \ k \leftrightarrow \frac{1}{C}$$
Consider two equations which correspond physical laws:

\[ \sum F = 0, \text{ Point mass} \]
\[ \sum \ddot{x} = 0, \text{ mechanical loop} \]

The analogous electrical laws are

\[ \sum V = 0, \text{ electrical loop} \]
\[ \sum i = 0, \text{ circuit node} \]

We MUST equate a point mass with an electrical loop and circuit node with a mechanical loop. In other words, we must map series mechanical connections to parallel electrical ones and vice versa.
Example 2

Convert the following mechanical system to an equivalent electrical network:

1) point mass = electrical loop

Example 2

2) A force generator is connected to the first mass. Thus we insert a voltage source in the first loop:

3) M1, M2 correspond to inductors. Each inductor should have a current which corresponds to the correct velocity therefore:
Example 2

4) $b, k$, are connected to both masses. The velocity/position which determines their forces is the difference between the two masses’ velocities. They thus correspond to resistance and capacitance connected into the common branch of the two loops since

$$\dot{x}_2 - \dot{x}_1 \rightarrow i_2 - i_1$$

where $R=b$, and $C=1/K$

Example 3: Contact

- Discontinuous contact is harder to model, but more important since contact always begins with and impact between the robot and environment. Consider a robot which is predominantly an inertia. Contact with a rigid wall could be modeled by a switch:

open: Contact $\rightarrow i_1 = 0$
closed: free motion $\rightarrow i_1 \neq 0$
Example 3: Contact

or, for a non-rigid environment:

$$L_E$$

Robot

$$i_1$$

Environment

$$R_E$$

$$C_E$$

The switch can be controlled by the position: $$\int_0^t i_1(t)dt$$

Electrical Conversion

Contact point = Circuit port
Two-port Network

Teleoperation system have two-contact point
Thus, two-circuit port

Correspondence btw. Mech. Elec.

velocity of the master arm \( \dot{x}_m \) \leftrightarrow \text{current} \( I_m \)
velocity of the slave arm \( -\dot{x}_s \) \leftrightarrow \text{current} \( I_s \)
operator’s force \( f_{op} \) \leftrightarrow \text{voltage} \( V_{op} \)
force at the master side \( f_m \) \leftrightarrow \text{voltage} \( V_m \)
force at the slave side \( f_s \) \leftrightarrow \text{voltage} \( V_s \)
Two-port Mapping

The relationship between efforts and flows is commonly described in terms of an immittance matrix \( P \).

Immittance mapping: \( y = Pu \)

**Impedance matrix**

\[
[f_m] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} v_m \\ -v_s \end{bmatrix}
\]

**Admittance matrix**

\[
[v_m] = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} f_m \\ -f_s \end{bmatrix}
\]

**Hybrid matrix**

\[
[f_m] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} v_m \\ -v_s \end{bmatrix}
\]

**Alternate hybrid matrix**

\[
[v_m] = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} f_m \\ -f_s \end{bmatrix}
\]

All of the immittance mapping satisfy the following condition

\[
y^T u = f_m v_m - f_s v_s
\]

Hybrid Parameter Interpretation

\[
h_{11} = \frac{v_m}{f_m} \bigg|_{v_s = 0} \quad \leftrightarrow \quad \text{Free motion input impedance}
\]

\[
h_{12} = \frac{v_m}{v_s} \bigg|_{f_s = 0} \quad \leftrightarrow \quad \text{Force feedback gain} \; \lambda_f
\]

\[
h_{21} = \frac{f_s}{v_m} \bigg|_{v_s = 0} \quad \leftrightarrow \quad \text{Forward velocity gain} \; -\lambda_p
\]

\[
h_{22} = \frac{f_s}{v_s} \bigg|_{f_m = 0} \quad \leftrightarrow \quad \text{Output admittance w/ clamped input}
\]

\[
H = \begin{bmatrix} Z_{In} & \lambda_f \\ -\lambda_p & 1/Z_{Out} \end{bmatrix}
\]
Dynamic Model of Haptic Interfaces

1. Mechanical Model
2. Electrical Model
3. Discrete Model with ZOH

Haptic Interaction System Overview
Mechanical and Electrical Model

![Diagram of mechanical and electrical model]

Human Operator + Haptic Interface + Virtual Environment

```
\[ m \ddot{v}_a + b v_a = f_h - f_a, \quad v_a = v_h \]
\[
\begin{bmatrix}
  f_h \\
  -v_a
\end{bmatrix} =
\begin{bmatrix}
  ms + b & 1 \\
  -1 & 0
\end{bmatrix}
\begin{bmatrix}
  v_h \\
  f_a
\end{bmatrix}
\]
```

Mechanical Model

```
\[ m \ddot{v}_a + b v_a = f_h - f_a, \quad v_a = v_h \]
```

```
\[
\begin{bmatrix}
  f_h \\
  -v_a
\end{bmatrix} =
\begin{bmatrix}
  ms + b & 1 \\
  -1 & 0
\end{bmatrix}
\begin{bmatrix}
  v_h \\
  f_a
\end{bmatrix}
\]```
Discrete Model with ZOH

\[ Z_d(z) = (ms + b) \left[ \frac{e^{-sT}}{s} \right] \rightarrow \left[ \frac{z-1}{T(z+1)} \right] \]

\[ \text{ZOH}(z) = \frac{1}{2} \frac{(z+1)}{z} \]

\[
\begin{bmatrix}
    f_h \\
    -v_c^*
\end{bmatrix}
= \begin{bmatrix}
    Z_d(z) & \text{ZOH}(z) \\
    -1 & 0
\end{bmatrix}
\begin{bmatrix}
    v_h \\
    f_c^*
\end{bmatrix}
\]