ME2025 Digital Control

Z Transform in State Space

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State-Space Representations of Discrete-time Systems

- Many ways to realize state-space representation
  - Controllable canonical form
  - Observable canonical form
  - Diagonal canonical form
  - Jordan canonical form

Z Transform Applied to State Space Models

Given

$$x[k+1] = Ax[k] + Bu[k]$$
$$y[k] = Cx[k] + Du[k]$$
$$x \in \Re^n \quad u \in \Re^m \quad y \in \Re^p$$

take the one-sided Z transform to get

$$zX(z) - zx[0] = AX(z) + BU(z)$$
$$Y(z) = CX(z) + DU(z)$$

$$X(z) = (zI - A)^{-1} zx[0] + (zI - A)^{-1} BU(z)$$

$$Y(z) = C[(zI - A)^{-1} zx[0] + (zI - A)^{-1} BU(z)] + DU(z)$$

$$Y(z) = C(zI - A)^{-1} zx[0] + C(zI - A)^{-1} BU(z) + DU(z)$$

Response from initial conditions　　　Response to input

We can rewrite the transfer function, using the formula for a matrix inverse

$$M^{-1} = \frac{adj(M)}{\det(M)}$$

$$H(z) \equiv C(zI - A)^{-1} B + D$$
$$= \frac{C\,adj(zI - A)B + D\det(zI - A)}{\det(zI - A)} = \frac{N(z)}{\det(zI - A)}$$

- det(zI-A) is an n^{th} order polynomial in z (note: not a matrix)

Poles of H(z) are values of z that make H(z) blow up---these are roots of the characteristic equation det(zI-A) = 0 which are the same as the eigenvalues of the A matrix

So stability of H(z) depends only on eigenvalues of A matrix