구조적 불확실성을 갖는 시스템의 Descriptor 형태의 표현을 통한 강인한 LQ 제어기 설계

Robust LQ Controller Design for Systems with Structured Uncertainty using Descriptor Form Representation

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Abstract : This paper introduced a new design method of Robust LQ regulator, which avoids the inversion of an inertia matrix allowing to use the full-structure information of structured time-varying parameter uncertainties.

The differential equation of system is expressed as a descriptor form to reserve the structure information of parameter uncertainties. Left-hand side matrix that has the uncertain parameters of inertia matrix is assumed to be Polytopic, and right-hand side matrices that include other uncertain parameters are treated as scaled small gain techniques such as μ -Synthesis.

For stability, the notion of quadratic stability, using a single Lyapunov function, is used. A set of conditions, under which the problem becomes convex, can be solved through finite-dimensional convex programming.

Keywords : Robust LQ regulator, structured parameter uncertainty, descriptor form, polytopic, scaled H_∞, LMI

I. INTRODUCTION

Design methods of a robust controller for uncertain systems using the given structure information have been given much attention recently. Many results have been obtained on norm-bounded uncertainty and the related problem of scaled H_∞ control [1]-[9]. In scaled H_∞ control, scaled small gain technique was used to use the given full structure information of uncertainties and reduce the conservatism. However, the bound of the uncertainties is unclear because a state space equation (A,B,C) with parameter perturbations is obtained through the inversion of the inertia matrix. If the inertia matrix has some uncertainty, conservative estimation of the bound of perturbations is inevitable owing to the matrix inversion. This causes the robust stabilizing controller to be too conservative [11].

In [8],[9], it is showed that the polytopic approach is effective to model the uncertainty, when the uncertainty enters in the model affinely. The search for the full state feedback law is equivalent to finding two matrices that satisfy a linear matrix inequality (LMI) at the vertices of the uncertainty polytope. We can obtain same solution to scaled H_∞ problem by applying a convex search for which efficient numerical methods are available [10]. However, polytopic approach needs as many LMI constraints as the number of the vertices of the uncertainty polytope. Thus, if the number of uncertain parameter increase, finding the increased vertices of the uncertainty polytope is very difficult and many constraints increase computation time. Thus, this is not a systematic approach, in this case.

In order to deal with the parameter uncertainties independently, μ -synthesis based on the descriptor form representation has been suggested in [11]. The descriptor form can represent differential equations of a system more naturally than the state-space form. In particular, for mechanical systems, independent physical parameters are preserved in the descriptor form. Especially, uncertainty structure on inertia matrix. Therefore, it is better to use the descriptor form as a representation of parameter uncertainty [11],[12].

To design the less conservative controller, a new method, that is using the Scaled H_∞ and polytopic approach, simultaneously is proposed. Represent differential equations of a system as a descriptor form and treat the left-hand side matrix which including the uncertainties of inertia term as a Polytopic and other right-hand side uncertain matrices as a scaled small gain technique. We can escape the conservatism, which arise at the inversion of inertia matrix. Therefore, it is possible to obtain a tight estimation of the bound of perturbations. Also, we can obtain the solution more easily and systematically using LMI.

This paper is organized as follows: the scaled H_∞ and polytopic approach is introduced in section II. Section III drives the main result that is minimization of the time domain performance index of LQ regulator with quadratic stability criteria. To evaluate the controller performance, the proposed control scheme is applied to the control of a single link flexible manipulator, which has time-varying structured parameter uncertainties in inertia matrix in section IV. Section V presents conclusions and further works.

II. PRELIMINARIES

First, polytopic approach and scaled small gain technique will be reviewed focusing on simple class of uncertain systems.

$$\dot{x} = A_\Delta x = (A_0 + \Delta A(t))x \quad (1)$$

$$z = Cx \quad (2)$$

The uncertainty matrix $\Delta A(t)$ is assumed to be polytopic, i.e., they depend linearly or affinely on the time varying parameters $\delta_i(t)$, $i = 1,\cdots,r$. Then, the uncertainty in the system (1) can be expressed as follows, without loss of generality.

$$\Delta A(t) = \sum_{i=1}^{r} \delta_i(t) A_i = M_A \Delta(t) N_A \quad (3)$$

The matrix A_i has a given uncertainty structure, and matrices M_A and N_A are used to describe the structure of uncertainty. For simplicity, we shall also assume that the uncertainty matrix $\Delta(t) \in \Lambda$ is real time-varying and

$$\Lambda = \left\{ block\_diag\left[\delta_1(t) I_{q_1} \quad \cdots \quad \delta_r(t) I_{q_r}\right] : \underline{\delta}_i \le \delta_i(t) \le \overline{\delta}_i \right\}.$$

For future reference, we shall denote the vertex set of Λ with the extreme values:

$$\Lambda_{vex} = \left\{ block\_diag\left[\delta_1(t) I_{q_1} \quad \cdots \quad \delta_r(t) I_{q_r}\right] : \delta_i(t) = \underline{\delta}_i \ or \ \delta_i(t) = \overline{\delta}_i \right\}.$$

It is easy to see that there are 2^r vertices in Λ_{vex} .

Lemma 1: The system described with equation (1) and (2) is quadratically stable if there exists a symmetric matrix $P > 0$ such that

$$A_\Delta^T P + P A_\Delta + C^T C < 0 \quad (4)$$

for all $\Delta(t) \in \Lambda$.

The above lemma is straightforward. The following lemma is polytopic approach in quadratic stability sense. This lemma is equivalent to the theorem 6 of [8].

Lemma 2: Consider the uncertain system described by equation (1) and (2). Then the following statements are equivalent:

i. There exists a symmetric matrix $P > 0$ such that

$$A_\Delta^T P + P A_\Delta + C^T C < 0$$

for all $\Delta(t) \in \Lambda$.

ii. There exists a symmetric matrix $P > 0$ such that

$$A_\Delta^T P + P A_\Delta + C^T C < 0$$

for all $\Delta(t) \in \Lambda_{vex}$.

Proof. The proof for i $\Longrightarrow$ ii is trivial since $\Lambda_{vex} \subset \Lambda$. To show ii $\Longrightarrow$ i, define $Q_\Delta \equiv A_\Delta^T P + P A_\Delta + C^T C$. Since $\Delta(t)$ appears affinely in Q_Δ , it is easy to see by convexity that

$$\max_{\Delta \in \Lambda} \lambda_{max}(Q_\Delta) = \max_{\Delta \in \Lambda_{vex}} \lambda_{max}(Q_\Delta) .$$

This implies that $Q_\Delta < 0 \quad \forall \Delta \in \Lambda$ if and only $Q_\Delta < 0 \quad \forall \Delta \in \Lambda_{vex}$.■

We shall also assume that the uncertainty is normalized so that $\overline{\delta}_i = -\underline{\delta}_i = 1$, i.e. $\|\Delta\| \le 1$. Factorization in (3) is not unique because of the existence of scaling matrices defined by the set

$$S_{\Gamma_A} \equiv \left\{ \Gamma_A \middle| \Gamma_A = blockdiag[\Gamma_{A1} \ \cdots \ \Gamma_{Ar}], det(\Gamma_{Ai}) \ne 0, \Gamma_{Ai} \in \Re^{q_i \times q_i} \right\} . \quad (5)$$

Note $\Gamma_A \Delta(t) \Gamma_A^{-1} = \Delta(t)$ for any $\Gamma_A \in S_{\Gamma_A}$. By means of the set S_{Γ_A} , we can represent all the possible I/O factorization of a given uncertainty as follows [13]:

$$\Delta A(t) = (M_A \Gamma_A) \Delta(t) (\Gamma_A^{-1} N_A) \text{ for any } \Gamma_A \in S_{\Gamma_A} . \quad (6)$$

Lemma 3: By scaled small gain criteria, the conditions in lemma 2 are true if and only if there is a symmetric matrices $P > 0$ and $X_A \in S_{X_A} \left(\equiv \left\{ \Gamma_A \Gamma_A^T \middle| \Gamma_A \in S_{\Gamma_A} \right\} \right)$ such that

$$A_0^T P + P A_0 + C^T C + P M_A X_A M_A^T P + N_A^T X_A^{-1} N_A < 0 \quad (7)$$

Proof. To do that, we note that for any $\Gamma_A \in S_{\Gamma_A}$, we have

$$\left((M_A \Gamma_A) \Delta(t) (\Gamma_A^{-1} N_A) \right)^T P + P (M_A \Gamma_A) \Delta(t) (\Gamma_A^{-1} N_A)$$
$$\le P M_A X_A M_A^T P + N_A^T X_A^{-1} N_A \quad (8)$$

for all $\Delta(t) \in \Lambda$. Using iequalities (7) and (8), we have immediately

$$A_\Delta^T P + P A_\Delta + C^T C < 0 , \quad \forall \Delta(t) \in \Lambda . \qquad ■$$

III. MAIN RESULTS

Inevitably, system models have fixed or time-varying parameter uncertainties in stiffness, damping, input and inertia matrix by modeling error. Generally, the dynamic equation of time-varying parameter uncertain systems can be modeled as follows:

$$(M_0 + \Delta M)\ddot{q} + (D_0 + \Delta D)\dot{q} + (K_0 + \Delta K)q = (G_0 + \Delta G)u \quad (9)$$

where $\Delta M = \sum_{i=1}^{q_M} \delta_i(t) M_i$, $\Delta D = \sum_{j=1}^{q_D} \delta_j(t) D_j$, $\Delta K = \sum_{k=1}^{q_K} \delta_k(t) K_k$ and $\Delta G = \sum_{l=1}^{q_G} \delta_l(t) G_l$. The real number $\delta_i, \delta_j, \delta_k, \delta_l$ are uncertain, time-varying and without loss of generality, satisfy $|\delta_i| \le 1$, $|\delta_j| \le 1$, $|\delta_k| \le 1$, $|\delta_l| \le 1$. The matrixes M_i , D_j , K_k , G_l are given uncertain structure. Descriptor-form of uncertain dynamic equation (9) is as follows:

$$E_\Delta \dot{x} = (A_0 + \Delta A)x + (B_0 + \Delta B)u \quad (10)$$

$$z = Cx + Du \quad (11)$$

where $x = [q \quad \dot{q}]^T \in \Re^n$ is the state vector, $u \in \Re^m$ is the control input vector, Z is the controlled output vector.

$$E_\Delta = \begin{bmatrix} I & 0 \\ 0 & M_0 + \Delta M \end{bmatrix} , \quad A_0 = \begin{bmatrix} 0 & I \\ -K_0 & -D_0 \end{bmatrix} , \quad B_0 = \begin{bmatrix} 0 \\ G_0 \end{bmatrix} ,$$
$$\Delta A = \begin{bmatrix} 0 & 0 \\ -\Delta K & -\Delta D \end{bmatrix} , \quad \Delta B = \begin{bmatrix} 0 \\ \Delta G \end{bmatrix} .$$

Normally, the matrix E_Δ in the descriptor form (10) is assumed to be non-singular and uncertainty matrix ΔM is polytopic. We can also define the compact set Λ^E , vertex set Λ^E_{vex} and Δ^M in (3), for uncertain matrix ΔM .

The state-space form of (10) is as follows:

$$\dot{x} = (E_\Delta^{-1} A_0 + E_\Delta^{-1} \Delta A)x + (E_\Delta^{-1} B_0 + E_\Delta^{-1} \Delta B)u , \quad (12)$$

The LQ quadratic performance index is defined as $J_{LQ} \equiv E\left[\int_0^\infty z^T z dt \right]$, where z is controlled output, which was defined in (11).

Theorem 1: The uncertain system described by equation (12) is quadratically stablizable using full state feedback $u = -Gx$. If there exist symmetric matrices $Z > 0$, $X_A > 0$, $X_B > 0$ and proper dimensioned Y satisfying the following matrix inequality constraint. Moreover, the performance index is bounded by $tr[PX(0)]$

$$\begin{bmatrix} \begin{pmatrix} E_\Delta Z A_0^T + A_0 Z E_\Delta^T \\ - E_\Delta Y^T B_0^T - B_0 Y E_\Delta^T \\ + M_A X_A M_A^T \\ + M_B X_B M_B^T \end{pmatrix} & E_\Delta (CZ - DY)^T & E_\Delta Z N_A^T & E_\Delta Y^T N_B^T \\ (CZ - DY) E_\Delta^T & -I & & \\ N_A Z E_\Delta^T & & -X_A & \\ N_B Y E_\Delta^T & & & -X_B \end{bmatrix} < 0$$

for all $\Delta^M \in \Lambda^M{}_{vex}$

(13)

Proof. If there exist Lyapunov function $V(x) = x^T P x$ and satisfy the followings

$$\frac{dV(x)}{dt} + z^T z < 0 \quad (14)$$

for $\forall \Delta^M \in \Lambda^M$, then the system (12) is quadratically stable and the quadratic performance index is bounded by $tr[PX(0)]$, after some manipulation. Where $X(0) = E\left[x(0) x(0)^T \right]$ [10]. If we expand (14) using (12) and $u = -Gx$, then we can obtain the following quadratic stability constraint

$$\left(E_\Delta^{-1}(A_0 - B_0 G) \right)^T P + P E_\Delta^{-1}(A_0 - B_0 G) + (C - DG)^T (C - DG)$$
$$+ \left(E_\Delta^{-1} M_A \Gamma_A \Delta \Gamma_A^{-1} N_A \right)^T P + P E_\Delta^{-1} M_A \Gamma_A \Delta \Gamma_A^{-1} N_A$$
$$- \left(E_\Delta^{-1} M_B \Gamma_B \Delta \Gamma_B^{-1} N_B G \right)^T P - P E_\Delta^{-1} M_B \Gamma_B \Delta \Gamma_B^{-1} N_B G < 0$$

for all $\Delta^M \in \Lambda^M$.

(15)

We deal with the uncertainties of system and input matrices via I/O factorization technique (6) as follows:

$$E_\Delta^{-1} \Delta A = (E_\Delta^{-1} M_A \Gamma_A) \Delta (\Gamma_A^{-1} N_A) , \; E_\Delta^{-1} \Delta B = (E_\Delta^{-1} M_B \Gamma_B) \Delta (\Gamma_B^{-1} N_B) \quad (16)$$

Sequentially, multiply P^{-1} and E_i to the both side of (15),

Substitute $P^{-1} = Z$, $GP^{-1} = Y$ and apply uncertainty bounding technique (8) . Finally, quadratic stability criteria can be transformed LMI constraints as follows:

$$\begin{bmatrix} \begin{pmatrix} E_\Delta Z A_0^T + A_0 Z E_\Delta^T \\ - E_\Delta Y^T B_0^T - B_0 Y E_\Delta^T \\ + M_A X_A M_A^T \\ + M_B X_B M_B^T \end{pmatrix} & E_\Delta (CZ - DY)^T & E_\Delta Z N_A^T & E_\Delta Y^T N_B^T \\ (CZ - DY) E_\Delta^T & -I & & \\ N_A Z E_\Delta^T & & -X_A & \\ N_B Y E_\Delta^T & & & -X_B \end{bmatrix} < 0$$

for all $\Delta^M \in \Lambda^M$.

(17)

Since, Δ^M appears affinely in the above constraint. Thus, the inequality constraint (17) is equivalent the followings, by lemma 2.

$$\begin{bmatrix} \begin{pmatrix} E_\Delta Z A_0^T + A_0 Z E_\Delta^T \\ - E_\Delta Y^T B_0^T - B_0 Y E_\Delta^T \\ + M_A X_A M_A^T \\ + M_B X_B M_B^T \end{pmatrix} & E_\Delta (CZ - DY)^T & E_\Delta Z N_A^T & E_\Delta Y^T N_B^T \\ (CZ - DY) E_\Delta^T & -I & & \\ N_A Z E_\Delta^T & & -X_A & \\ N_B Y E_\Delta^T & & & -X_B \end{bmatrix} < 0$$

for all $\Delta^M \in \Lambda^M_{vex}$.

(18)

Thus we can say that the uncertain system (12) is quadratically stable, if we can find symmetric matrices $Z > 0$, $X_A > 0$, $X_B > 0$ and proper dimensioned Y such that satisfies matrix inequality condition (18) for all vertices in Λ^M_{vex} , simultaneously. Moreover, the full state feedback controller can be taken as $G = YZ^{-1}$.

■

Note that, to explain the equality condition $Z = P^{-1}$, inequality condition

$$Z - P^{-1} > 0 \quad \text{or} \quad \begin{bmatrix} Z & I \\ I & P \end{bmatrix} > 0 \qquad (19)$$

should be added.

This theorem shows that the design problem of robust LQ regulator can be reduced to searching for the matrices $Z > 0$, $X_A > 0$, $X_B > 0$ and proper dimensioned Y minimizing $tr[PX(0)]$ with satisfying the linear matrix inequality constraints (18), (19). This matrix inequalities are convex in Z , X_A , X_B and Y , thus convex programming techniques can be used to solve for Z , X_A , X_B and Y .

IV. NUMERICAL EXAMPLE

We consider the following example, which is based on the model for a single-link flexible manipulator. It's dynamic equation which is driven by continuous model is described as follows [14]:

$$[M]\ddot{q} + [D]\dot{q} + [K]q = [B]\tau \qquad (20)$$

$$q = \begin{Bmatrix} q_i \\ \vdots \\ q_n \end{Bmatrix} \quad \text{for} \quad i, j = 0, 1, , \cdots n$$

$$[M] = \begin{bmatrix} M_{ij} & \cdots \\ \vdots & \ddots \end{bmatrix}, \quad M_{ij} = \begin{pmatrix} \rho A \int_0^l \Phi_i(x) \Phi_j(x) dx + I_h \Phi_i'(0) \Phi_j'(0) \\ + M_e \Phi_i(l) \Phi_j(l) + J_e \Phi_i'(l) \Phi_j'(l) \end{pmatrix}$$

$$[D] = c_0 \begin{bmatrix} \Phi_i'(0) \Phi_j'(0) & \cdots \\ \vdots & \ddots \end{bmatrix}, \quad [B] = \begin{bmatrix} \Phi_i'(0) \\ \vdots \end{bmatrix}$$

$$[K] = \begin{bmatrix} 0 & 0 & \cdots \\ 0 & K_{ij} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \qquad K_{ij} = EI \int_0^l \Phi_i''(x) \Phi_j''(x) dx$$

Where $\Phi_{i,j}(\cdot)$: Mode function, EI : stiffness of link, ρA : unit length mass of link, L : length of link M_e : Tip mass, J_e : Tip rot. inertia and I_h : rot. inertia of Hub. Refer to [15], for more detail.

If we assume that, tip mass and Tip rot. inertia have 50% time-varying parameter uncertainty and stiffness of link can have 30% parameter variation, we can extract structured time varying parameter uncertainties.

$$[M + \Delta M]\ddot{q} + [D]\dot{q} + [K + \Delta K]q = [B]\tau \qquad (21)$$

Where

$$[\Delta M] = \begin{bmatrix} \Delta M_{ij} & \cdots \\ \vdots & \ddots \end{bmatrix},$$

$$\begin{aligned} \Delta M_{ij} &= \Delta M_e \Phi_i(l) \Phi_j(l) + \Delta J_e \Phi_i'(l) \Phi_j'(l) \\ &= \delta_1(t) \left(0.5 M_e \Phi_i(l) \Phi_j(l) + 0.5 J_e \Phi_i'(l) \Phi_j'(l) \right) \end{aligned}$$

$$[\Delta K] = \begin{bmatrix} 0 & 0 & \cdots \\ 0 & \Delta K_{ij} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \quad \begin{aligned} \Delta K_{ij} &= \Delta EI \int_0^l \Phi_i''(x) \Phi_j''(x) dx \\ &= \delta_2(t)(0.3EI) \int_0^l \Phi_i''(x) \Phi_j''(x) dx \end{aligned}$$

Notice that, ΔM_e and ΔJ_e are dependent uncertainties, thus, we can treat these as one parameter uncertainty $\delta_1(t)$. These uncertainties can be normalized so that $|\delta_1(t)| \le 1$, $|\delta_2(t)| \le 1$.

In this example, we consider to third flexible mode, that is, 4th order system is considered. By using the proposed design method, we can obtain the robust LQ regulator. This can be obtained by solving multi-constrained convex optimization problem.

To evaluate the robust performance of the controller, simulation results of proposed controller with fixed and time-varying parameter uncertainties are presented in Fig. 1 and Fig. 2. These figures show that the closed-loop system can be stable in the face of parameter variation.

To show the effectiveness of the designed controller, comparison of nominal LQ regulator and proposed controller without uncertainty is shown in Fig. 3. These figures show the nonconservatism of this proposed design method.

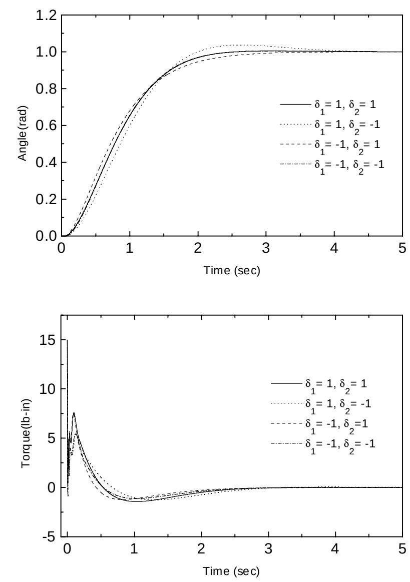

Fig. 1. Robust Performance of fixed uncertainty (Step response)

V. CONCLUSIONS

This paper has proposed a new design method of Robust LQ regulator based on the descriptor form of structured time-varying uncertain systems to avoid the inversion of inertia matrix, and this method has been applied to the vibration control of single-link flexible manipulator. The resulting performance of the control system is satisfactory.

A quadratic stability criterion, which is based on polytopic approach and scaled small gain technique, is presented. this design problem can be formulated as a convex programming problem and gives, in general, less conservative results than those obtained using only the scaled small gain technique. Moreover, it's more practical than that using only polytopic approach.

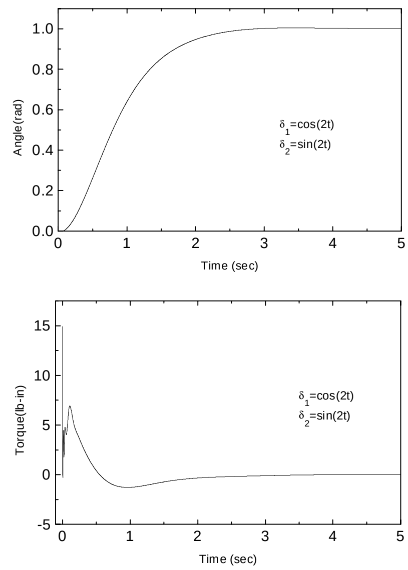

Fig. 2. Robust Performance of time-varying uncertainty (Step response)

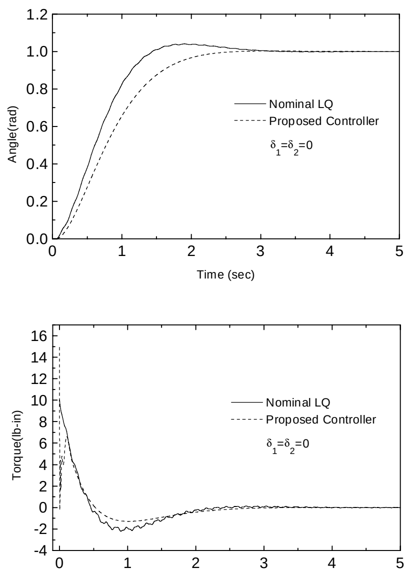

Fig. 3. Nominal Performance (Step response)

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