Sampled- and Continuous-Time Passivity and Stability of Virtual Environments

Jee-Hwan Ryu, Yoon Sang Kim, and Blake Hannaford

Abstract—We propose a new time-domain passivity observer (PO) and passivity controller (PC) which removes the constant-velocity assumption during one sample time, which was used in our previous PO/PC approach. A new sampled-time definition of passivity is introduced, and this new definition is compared with the previous sampled-time definition of passivity. Through this comparison, we propose the more accurate PO/PC approach. The proposed new PO/PC approach is applied to the "Excalibur" haptic interface system with very high stiffness (K = 120 kN·m) virtual environment, and stable contact is demonstrated.

Index Terms—Haptic interface, passivity controller (PC), passivity observer (PO), time-domain passivity.

I. INTRODUCTION

A haptic interface is a kinesthetic link between a human operator and a virtual environment (VE). One of the most significant problems in haptic interface design is to create a control system which is simultaneously stable and gives high fidelity under any operating conditions and for any VE parameters. There are several mechanisms by which a VE or other part of the system might exhibit active behavior. These include quantization [4], interactions between the discrete-time system and the continuous-time device/human operator [5], and delays due to numerical integration schemes [8]. These contributing factors to instability have been termed "energy leaks" by Gillespie and Cutkosky [6]. The most common approach to this problem is to add damping to the VE system and/or reduce the maximum stiffness which can be rendered. Thus, an engineering tradeoff is presented, since realism of the haptic interface (for example, in terms of stiffness of "hard" objects) must often be reduced in order to guarantee totally stable operation.

Initial efforts to solve this problem introduced the "virtual coupling" between the VE and the haptic device [4], [10]. The virtual coupling parameters can be set empirically, but several previous research projects have sought out a theoretical design procedure using control theory. However, interesting VEs are always nonlinear, and the dynamic properties of a human operator are always involved. These factors make it difficult to analyze haptic systems in terms of system models with known parameters and linear control theory. One fruitful approach is to use the idea of passivity to guarantee stable operation. Haptic interfaces consist of a human operator, haptic device, and VE. If we can make the VE ideally passive, haptic interfaces will be passive and stable, since the connected haptic device and human can be considered passivity ideas in the related area of stable control of force-feedback teleoperation with time delay. Colgate and Schenkel [5] and Adams and Han-

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naford [1] have used it to derive fixed-parameter virtual couplings (i.e., haptic interface controllers).

The major problem with using passivity for the design of haptic interaction systems is that it is overconservative. In many cases, performance can be poor if a fixed damping value is used to guarantee passivity under all operating conditions.

Recently, a different passivity-based approach has been proposed by Hannaford and Ryu [7] that measures active system behavior and injects variable damping without any knowledge of model information. They proposed a "passivity observer" (PO) and a "passivity controller" (PC) to insure stable contact under a wide variety of operating conditions. The PO can measure energy flow in and out of one or more subsystems in real time, and the PC is an adaptive dissipative element containing a series (velocity conserving) or parallel (force conserving) element interposed between the haptic interface and the VE. It absorbs exactly the net energy output (if any) measured by the PO at each time sample.

In their previous research [7], they have assumed that the continuous-time velocity is constant during one sample, based on the assumption that the sampling rate is sufficiently fast, compared with system dynamics. Because of this, our previous PC was able to make the network passive *before* the energy was produced. However, if the system dynamics are not slow enough to satisfy the constant-velocity assumption, the error could accumulate with the numerical integration. Thus, the earlier PO could not observe active behavior of a VE in certain cases. Recently, Stramigioli introduced the case where the velocity is not constant during one sample to show that there is no energy loss between a discrete-time VE and a continuous-time haptic interface at the sampling times [11].

In this paper, we propose a more accurate PO and PC approach, considering velocity changes during one sample time. A modified definition of sampled passivity, based on [11], is introduced, and it is compared with the former definition [7]. With combining the previous PO and the new PO based on the modified sampled-time definition of passivity, we propose a new PO and PC approach. We analyze and experimentally verify the passivity and stability of a haptic interface, considering not only sampled time passivity, but also continuous-time passivity. In Section II, we review passivity properties of networks in continuous-time, and introduce a sampled-time definition of passivity. Section III proposes a more accurate time-domain passivity-control approach, based on the sampled-time definition of passivity (5) which can measure exact energy output. Energy behavior between samples is analyzed in Section IV. Section V shows several experimental results. The conclusion follows in Section VI.

II. PASSIVITY IN CONTINUOUS AND SAMPLED TIME

The sign convention for all forces and velocities is defined so that their product is positive when power enters the system port (Fig. 1). Also, the system is assumed to have initial stored energy at t = 0 of E(0). The following widely known definition of passivity is then used.

Definition 1: The one-port network N with initial energy storage E(0) is continuous-time passive if and only if

$$\int_{0}^{t} f(\tau)\dot{x}(\tau)d\tau + E(0) \ge 0 \quad \forall t \ge 0$$
(1)

for force f and velocity \dot{x} . Equation (1) states that the energy supplied to a passive network must be greater than negative E(0) for all time [12], [13].



Fig. 1. One-port network model.



Fig. 2. Sampled time notation.

The elements of a typical haptic interface system include the VE, the virtual coupling network, the haptic device controller, the haptic device, and the human operator. Many of the input and output variables of these elements of haptic interface systems can be measured by computer, and the conjugate variables which define power flow in such a computer system are sampled time values. The haptic interface or tele-operator system is assumed to take a position as an input, and computes force as its output. Typically, this position input comes from position sensors such as encoders, and the computed force value is applied to the environment and/or operator through motors controlled directly by the output of a zero-order hold (ZOH).

Several variables are defined for the sampled time system (Fig. 2) during one sample time, $[t_{k-1} \le t \le t_k]$.

- 1) $f(t) = F(t_{k-1})$ is the constant force output of the ZOH.
- 2) $\dot{x}(t)$ is the system velocity, which is not constant.

3) $x(t_k)$ and $x(t_{k-1})$ are the position at k and k-1 sample times. The energy output during this single sample interval [11] is

$$\int_{t_{k-1}}^{t_k} F(t_{k-1})\dot{x}(\tau)d\tau = F(t_{k-1})\left(x(t_k) - x(t_{k-1})\right).$$
(2)

Summing over all samples, the total energy output at each sample time is

$$\sum_{j=0}^{k} F(t_{j-1}) \left(x(t_j) - x(t_{j-1}) \right)$$
(3)

and the continuous-time energy output and sampled-time energy output are equivalent

$$\int_{0}^{t_{k}} f(\tau)\dot{x}(\tau)d\tau = \sum_{j=0}^{k} F(t_{j-1})\left(x(t_{j}) - x(t_{j-1})\right)$$
(4)

where k = 0, 1, 2, ... Using (4), we can exactly measure the energy flow from the sampled time VE to the continuous system (such as a haptic device) at each sampling time, independent of sampling frequency. Based on the above fact, we can define a sample-time passivity.

Definition 2: The one-port network N with initial energy storage E(0) is sampled time passive if and only if

$$E_{\rm ST}(t_k) = \sum_{j=0}^k F(t_{j-1}) \left(x(t_j) - x(t_{j-1}) \right) + E(0) \ge 0 \quad (5)$$

where k = 0, 1, 2, ..., for sampled force $F(t_j)$ and position $x(t_j)$.

If $E_{ST}(t_k) \ge 0$ for every k, this means the system dissipates energy. If there is an instance that $E_{ST}(t_k) < 0$, this means the system generates energy, and the amount of generated energy is $-E_{ST}(t_k)$.

A. Comparison With Previous PO

In our previous paper [7], system stability was analyzed in terms of a PO equivalent to the following definition of sampled passivity:

$$\sum_{j=0}^{k} F(t_{j-1}) \left(x(t_{j-1}) - x(t_{j-2}) \right) + E(0) \ge 0.$$
(6)

This definition was derived from the continuous-time definition of passivity (1), assuming that the sampling rate is substantially faster than the dynamics of the haptic device, human operator, and VE. Equivalently, we assumed the change in velocity within one sample is very small. During one sample time, $[t_{k-1} \le t \le t_k]$, $F(t_{k-1})$ was the constant force output, and we assumed that the velocity input also has the constant value $((x(t_{k-1}) - x(t_{k-2}))/\Delta T)$, and the backward velocity-estimation value at t_{k-1} , because it was required to predict the one-step-ahead energy output.

On the other hand, in our new definition, (5), we calculate the energy output considering the velocity change within one sample. Thus, exact energy output can be measured from the VE. However, we only know the amount of generated energy *after* energy comes out, since we do not know the future position displacement $x(t_k)$. The previous PO predicted the amount of produced energy from t = 0 to $t = t_k$ at time t_{k-1} , before the energy is produced. If the system dynamics are significantly slow compared with the sampling rate, (5) and (6) will have almost the same values. However, if the system dynamics are not slow enough, the numerical integration error caused by the constant-velocity assumption could increase.

III. NEW PC SCHEME

A. PO

The new sampled-time definition of passivity can measure the exact energy output from a VE at each sampling time, but it can only measure the amount of energy output after the energy is already produced, so it can not avoid the active behavior. To try to obtain the prediction ability of the original PO with the accuracy of the new formulation, we combine the above two ideas and define a new PO as follows:

$$W_c(k+1) = W(k) + F(t_k)(x(t_k) - x(t_{k-1}))$$
(7)

where

$$W(k) = W(k-1) + F(t_{k-1})(x(t_k) - x(t_{k-1})).$$
(8)

W(k) is the new sampled-time definition of passivity measure, which is the total energy output from 0 to t_k , and $W_c(k)$ is the new PO which combines the new sampled-time passivity measure and one-step-ahead energy prediction. The last term of (7) is the estimation of the one-step-ahead energy output, which is the output energy from t_k to t_{k+1} , based on the assumption that the velocity during one sample $[t_k \leq t \leq t_{k+1}]$ will be constant. This is the same assumption as the previous PO. However, in this case, the error caused by this assumption is not integrated.

B. PC

Based on the newly developed PO (7) and steps 4 and 5 below, the PC algorithm (steps 6 and 7 below) for a one-port network with impedance causality (Fig. 3) is similar to an earlier paper [7]:

- 1) $x_1(k) = x_2(k)$ is the input;
- 2) $\Delta x(k) = x_1(k) x_1(k-1);$
- 3) $f_2(k)$ is the output of the one-port network;



Fig. 3. One-port network with PC.

- 4) $W(k) = W(k-1) + f_1(k-1)\Delta x(k)$ is the energy output at step k;
- 5) $W_c(k+1) = W(k) + f_2(k)\Delta x(k)$ is the prediction of the energy level at step k + 1;
- 6) the PC control force to dissipate the produced energy is calculated

$$f_{\rm PC}(k) = \begin{cases} \frac{-W_c(k+1)}{\Delta x(k)}, & \text{if } W_c(k+1) < 0\\ 0, & \text{if } W_c(k+1) \ge 0 \end{cases}$$
(9)

7) $f_1(k) = f_2(k) + f_{PC}(k)$ is the output.

For the developed passivity-control scheme, we can prove the stability by showing the passivity with PC $(W(k + 1) \ge 0 \forall k)$ since passivity is the sufficient condition of the stability. At the k + 1 step, the actual energy output value will be

$$W(k+1) = W(k) + f_1(k)\Delta x(k+1).$$
 (10)

After simple mathematical manipulation, (10) can be represented as follows when the $f_{PC}(k) \neq 0$:

$$W(k+1) = W(k) \left(1 - \frac{\Delta x(k+1)}{\Delta x(k)}\right). \tag{11}$$

Generally, the energy level (11) will stay near zero $(W(k + 1) \approx 0)$ since the velocity change is small $(\Delta x(k + 1)/\Delta x(k) \approx 1)$ within one sample. And in most of the cases, the energy output goes back to a positive value even though it was negative, which means energy delivered to the device/operator attached to the VE will be dissipated. The only case where the energy level W(k + 1) stays below zero for all the time $(W(k + 1) < 0 \forall k)$ is if:

- 1) W(k) < 0;
- 2) $\operatorname{sign}(\Delta x(k+1)) = \operatorname{sign}(\Delta x(k));$
- 3) $|\Delta x(k+1)| < |\Delta x(k)|;$

are all satisfied. However, these conditions mean that the velocity output of the system is gradually decreased, and the net energy output of the system approaches zero, which means that physically, the system is stable. The only exception occurs if $\operatorname{sign}(\Delta x(k+1)) = -\operatorname{sign}(\Delta x(k))$ for all k. But, since we assume that sampling rate is sufficiently faster than system modes, the above behavior can not happen without heavy noisy effects.

IV. ENERGY BEHAVIOR BETWEEN SAMPLES

Even though by using (8) we can measure the exact energy output from a VE at each sampling time, and can make the system sampled-time passive using the developed time-domain PO/PC, sampled-time passivity is not equivalent to continuous-time passivity. For a VE with impedance causality, the force output from the environment is constant during one sample, due to ZOH. However, we do not assume the velocity input is constant. Thus, we may have generally three kinds of energy behavior in between samples, depending on three kinds of velocity profiles qualitatively illustrated in Fig. 4. In the following, we study these possible evaluations of the system dynamics with the worst-case assumption that the value of discrete-time and continuous-time POs are zero at $t = t_{k-1}$.



Fig. 4. Energy behavior with three kinds of velocity profiles.

First, for the case of continuous passive and sampled passive (Fig. 4, $\dot{x}_a(t)$), although power (the product of force and velocity) is briefly negative, the energy value remains positive $(E_a(t))$ during the entire interval, which means there is no net energy generation. As a result, the closed-loop system remains stable.

When the VE is continuous active and sampled active, in this case $(\dot{x}_b(t))$, the velocity changes sign and total energy is produced $(E_b(t) < 0)$ at $t = t_k$. As a result, the system is continuous-time active and sampled active, as well. In this case, we need to apply the developed PO/PC approach to make the system stable by compensating the produced energy.

Finally, when the velocity profile is like $\dot{x}_c(t)$, which is continuous active and sampled passive, power and energy go negative between sample times, but are positive again $(E_c(t))$ at $t = t_k$. Even though the system is not passive in continuous time anymore, since the actual energy output goes to a negative value between sample times, the PO could not monitor the active behavior. In this case, we need to investigate whether this system is stable or not in continuous time without any PC action. Intuitively, although the system produces energy, the produced active amount of energy is dissipated by the system within one sampling time. Thus, we can say that the system does not diverge. In other words, since the systems connected to the VE are passive systems, such as the mechanism of the haptic device and human operator, the output will be bounded for a bounded input. As a result, the system can be stable in terms of bounded-input/bounded-output (BIBO) stability, and we do not need to dissipate energy artificially in this case. Although this result demonstrates one kind of stability, an important type of instability can still arise, which results in "small" displacements or oscillations which are still perceptible to the user.

V. EXPERIMENTAL RESULTS

We first test the previous PO/PC approach [7] for the high stiffness $(K = 120 \text{ kN} \cdot \text{m})$ VE with Excalibur haptic interface (in the previous paper [7], we used $K = 90 \text{ kN} \cdot \text{m}$). Even though the *measured* energy level (PO value) remains positive [Fig. 5(c)], the system is unstable, resulting in an oscillation [Fig. 5(a), (b)]. Because of the higher stiffness,



Fig. 5. Contact response with the previous PO/PC for high-stiffness VE K = 120 kN·m.



Fig. 6. New sampled-time passivity measure value for the experiment of Fig. 5.

the dynamics of the VE are now faster than the case of K = 90 N·mm. Thus, as described in Section III, the previous PO could not observe the active amount of energy, since the constant-velocity assumption during one sample time was not satisfied.

Using the recorded data from this experiment (Fig. 5), we calculated the new sampled time passivity measure (5) (Fig. 6). The new passivity measure becomes negative after the first bounce, indicating that it correctly measures the active behavior. Since the previous PO could not measure the exact amount of produced energy due to the



Fig. 7. Contact response with the sampled-time passivity measure as a PO and PC for high-stiffness VE K = 120 kN·m.

constant-velocity assumption, the PC did not dissipate the produced energy.

We then applied the PC using the sampled-time passivity measure (5) as the PO. Now we could stabilize the system with about four bounces [Fig. 7(a) and (b)]. However, the actual energy value crossed down to negative values at the end of each bounce [Fig. 7(c)]. Since we can only know how much energy is produced after energy is already produced with the sampled-time passivity measure (5), we could not avoid active energy behavior for several sampling times, even though the system showed stable behavior. In this case, the total system could be stabilized, since the haptic device and human operator had enough damping to dissipate the brief active behavior. However, if a device has not enough damping, compared with the amount of active behavior that we are misssing, the system will oscillate. We obtained several experimental results of this type with impedance-type haptic devices in our laboratory having lower damping levels.

We applied the proposed new PO/PC, (7) and (9), for the same experimental conditions. Similar stable contact with Fig. 7 was achieved. However, since the new PO predicted the future energy output, we could make the actual energy value stay above zero (Fig. 8), while the actual energy of the above experiment crossed down to negative value [Fig. 7(c)].

Fig. 8. Actual energy behavior with the new PO/PC approach for high-stiffness VE K = 120 kN·m.

VI. CONCLUSION

In this paper, a more accurate time-domain passivity-control approach is proposed, considering the velocity change during one sample time. The actual energy output can be measured precisely with the sampled-time passivity measure, but we can only know the actual energy output after energy is already produced. To avoid the active behavior, we proposed a new PO, combining both predictive and accurate features, and designed the PC based on the new PO. We analyzed the sampled- and continuous-time energy behavior, and proved that the sampled passive system is at least stable, even though it is not passive in continuous time. The experiments showed that we could achieve stable contact with a higher virtual spring stiffness than by using the earlier technique.

REFERENCES

- R. J. Adams and B. Hannaford, "Stable haptic interaction with virtual environments," *IEEE Trans. Robot. Automat.*, vol. 15, pp. 465–474, June 1999.
- [2] —, "Excalibur, a three-axis force display," in Proc. ASME Winter Annu. Meeting Haptics Symp., Nashville, TN, Nov. 1999, pp. 289–296.
- [3] R. J. Anderson and M. W. Spong, "Asymptotic stability for force reflecting teleoperators with time delay," *Int. J. Robot. Res.*, vol. 11, no. 2, pp. 135–149, 1992.
- [4] J. E. Colgate, M. C. Stanley, and J. M. Brown, "Issues in the haptic display of tool use," in *Proc. IEEE/RSJ Int. Conf. Intelligent Robotics* and Systems, Pittsburgh, PA, 1995, pp. 140–145.
- [5] J. E. Colgate and G. Schenkel, "Passivity of a class of sampled-data systems: Application to haptic interfaces," in *Proc. American Control Conf.*, Baltimore, MD, 1994, pp. 3236–3240.
- [6] B. Gillespie and M. Cutkosky, "Stable user-specific rendering of the virtual wall," in *Proc. ASME Int. Mechanical Engineering Conf. Expo.*, Atlanta, GA, Nov. 17–22, 1996, pp. 397–406.
- [7] B. Hannaford and J. H. Ryu, "Time-domain passivity control of haptic interfaces," *IEEE Trans. Robot. Automat.*, vol. 18, pp. 1–10, Feb. 2002.
- [8] B. E. Miller, J. E. Colgate, and R. A. Freeman, "Environment delay in haptic systems," in *Proc. IEEE Int. Conf. Robotics and Automation*, San Francisco, CA, Apr. 2000, pp. 2434–2439.
- [9] G. Niemeyer and J. J. Slotine, "Stable adaptive teleoperation," *IEEE J. Oceanic Eng.*, vol. 16, pp. 152–162, Feb. 1991.
- [10] C. B. Zilles and J. K. Salisbury, "A constraint-based God-object method for haptic display," in *Proc. IEEE/RSJ Int. Conf. Intelligent Robotics and Systems*, Pittsburgh, PA, 1995, pp. 146–151.
- [11] S. Stramigioli, C. Secchi, and A. J. van der Schaft, "A novel theory for sampled data system passivity," in *Proc. IEEE/RSJ Int. Conf. Intelligent Robotics and Systems*, Lausanne, Switzerland, 2002, pp. 1936–1941.
- [12] A. J. van der Schaft, L2-Gain and Passivity Techniques in Nonlinear Control, ser. Communications and Control Engineering. New York: Springer, 2000.
- [13] J. C. Willems, "Dissipative dynamical systems, Part I: General theory," *Arch. Rat. Mech. Ann.*, vol. 45, pp. 321–351, 1972.

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Control of a Flexible Manipulator With Noncollocated Feedback: Time-Domain Passivity Approach

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Abstract—A new method to control a flexible manipulator with noncollocated feedback is proposed. We introduce a method to implement the timedomain passivity-control approach to a flexible manipulator with noncollocated feedback, which could not be treated with the previous time-domain passivity-control framework due to a possible active transfer function from the input to the noncollocated output. The proposed method is simulated with a single-link flexible manipulator, and a good control performance is obtained.

Index Terms—Flexible manipulator, noncollocated feedback, passivity controller (PC), passivity observer (PO), time-domain passivity.

I. INTRODUCTION

Flexible manipulators are finding their way into industrial and space robotic applications due to their lighter weight and faster response time, compared with rigid manipulators. Control of flexible manipulators has been studied extensively for more than a decade by many researchers [2], [3], [12], [20], [23], [25]. Despite their results, this control problem has proven to be rather complicated.

It is well known that stabilization of a flexible manipulator can be greatly simplified by collocating the sensors and the actuator, where the input–output (I/O) mapping is passive [26], and a stable controller can be easily devised independent of the structural details. However, the performance of this collocated feedback turns out to be unsatisfactory, due to a weak control of the vibrations of the link [4]. This initiated finding other noncollocated output measurements, such a position of the end-point of the link to increase the control performance [3]. However, if the end-point is chosen as the output and the joint torque is chosen as the input, the system becomes a nonminimum phase one, and may behave actively. As a result, a small increment of feedback controller gains can easily make the closed-loop system unstable. This led many researchers to seek other outputs which have the passivity property.

Wang and Vidyasagar proposed the so-called reflected tip position as such an output [26]. This corresponds to the rigid-body deflection minus the deflection at the tip of the flexible manipulator. Pota and Vidyasagar used the same output to show that in the limit, for a nonuniform link, the transfer function from the input torque to the derivative of the reflected tip position is passive whenever the ratio of the link inertia to the hub inertia is sufficiently small [15]. Chodavarapu and Spong considered the virtual angle of rotation, which consists of the hub angle of rotation augmented with a weighted value of the slope of the link at its tip [4]. They showed that the transfer function with this output is minimum phase and that the zero dynamics are stable.

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